

# Validation of generalized equivalent circuit's modeling: Shielding Application

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**Ahmed Nouainia, Mohamed Hajji, Taoufi Aguil**

Sys'Com Laboratory, National Engineering School of Tunis, Tunis El Manar university,  
B.P.37 Le Belvedere, Tunis 1002, Tunisia.

ahmed\_nouainia@yahoo.fr

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# Abstract

The electromagnetic analysis of planar circuits at high frequencies is based on several numerical modeling methods. Among those, method of generalized equivalent circuit (MGEC) is well adapted for studying planar circuits due to their multiple advantageous. However, when the studied structure complexity increases, it attains rapidly its limitation in terms of requirements (computational time and memory storage). It can be affected also by badly scaled matrices problem that constitutes a huge problem for all numerical methods. In this work, we are interested in optimizing the basic parameters of the MGEC method in order to avoid the considered previous problems. In fact, according the possible cases of excitation source and test function, we can obtain four possible configurations of (GEC) that can model an electromagnetic problem. Our aim is to compare between these configurations in terms of accuracy and needed time to reach convergence, and then determine the well GEC. This validated by applying the MGEC to study shielding. Primary results show that there are two unsuitable GEC configurations which are affected by numerical problems and the two others (classic GEC) are adequate. Thus, we had been interesting in avoiding these problems and remedy the two unsuitable configurations. This challenge is guaranteed by introducing the wave concept as excitation source to get a possible electromagnetic solution.

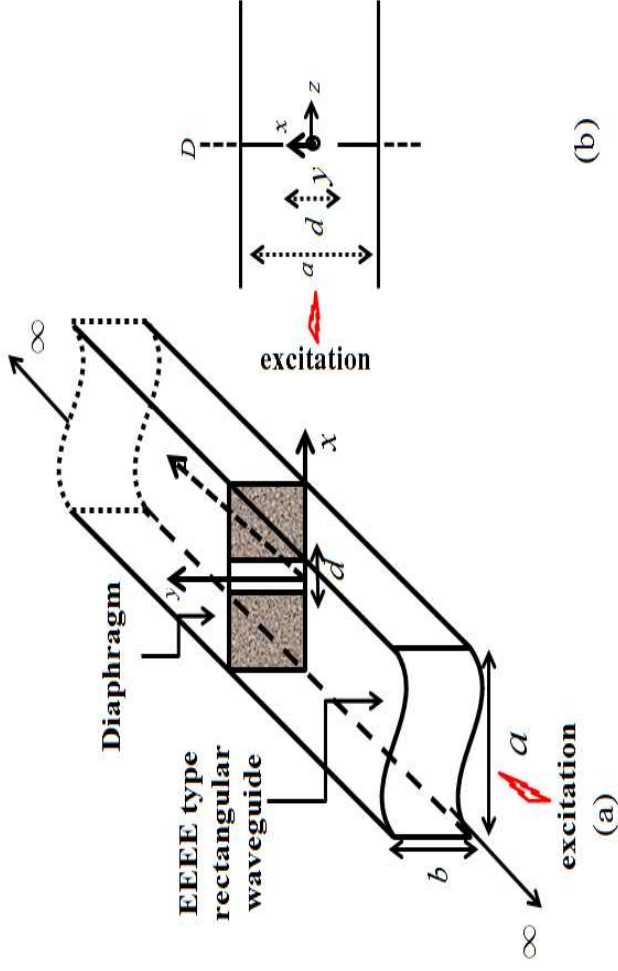
**Keywords:** Generalized Equivalent Circuit; electromagnetic problem; wave concept; shielding

# Biography

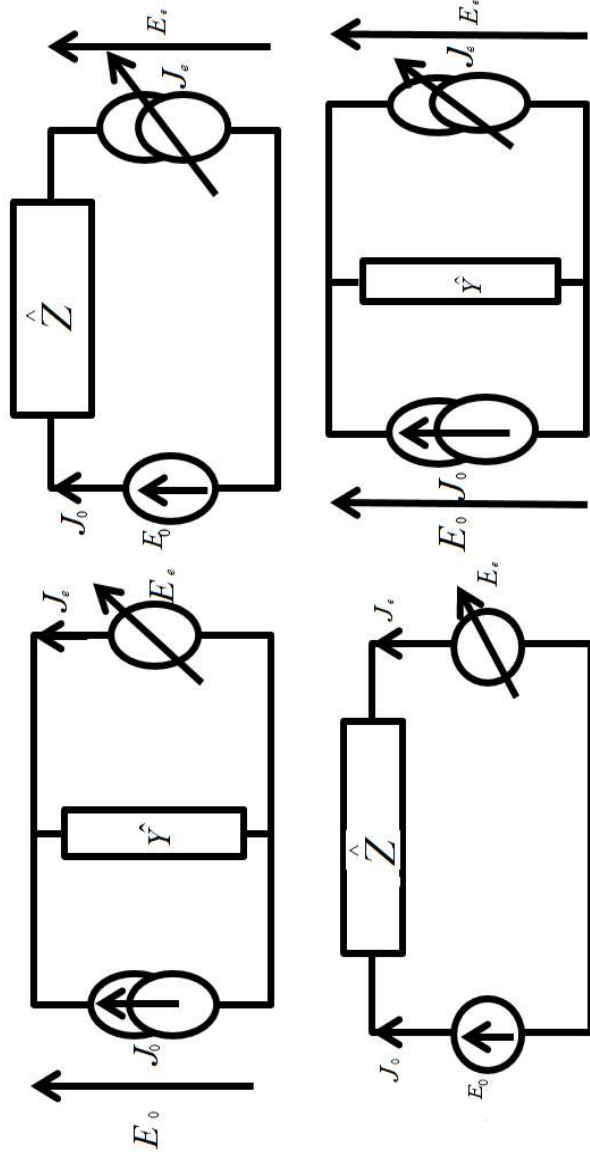


**Ahmed NOUAINIA** received the Master degree in physics of soft matter and the MSc degree in sciences physics from faculty of physical mathematics and natural sciences of Tunis (FST). He is a PhD student in physical at faculty of physical mathematics and natural sciences of Tunis. His research interest is in the electromagnetic modelling of complex structures using MoM methods for microwave and antenna applications.

# Validation MGEC



Equivalent circuits



# First GEC configuration

## Current (source)/Field (test function)

This version of the equivalent circuit is formed by a current excitation source  $J_0 = I_0 f_0$ , an admittance operator and a field virtual source  $E_e$  defined on the aperture domain of the discontinuity surface.  $E_e$  is the problem unknown approximated as a series of known test functions  $g_p$  weighted by unknown coefficients  $v_p$   $p = 1, 2, \dots, N_e$ ,  $E_e = \sum_p g_p v_p$

Application of Ohm and Kirchhoff laws: 
$$\begin{pmatrix} E_0 \\ J_e \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & \hat{Y} \end{pmatrix} \begin{pmatrix} J_0 \\ E_e \end{pmatrix}$$

Galrekin Method: 
$$\begin{pmatrix} V_0 \\ 0 \end{pmatrix} = \begin{bmatrix} 0 & \langle f_0 | g_p \rangle \\ -\langle g_q | f_0 \rangle & \langle g_q | \hat{Y} | g_p \rangle \end{bmatrix} \begin{pmatrix} I_0 \\ v_p \end{pmatrix} \Leftrightarrow \begin{pmatrix} V_0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 & A \\ -A^T & B \end{pmatrix} \begin{pmatrix} I_0 \\ v_p \end{pmatrix}$$

The resolution of this system allows to calculate the input impedance:

$$Z_{in} = \frac{V_0}{I_0} = A^T B^{-1} A$$

# Second GEC configuration:

## Field (source)/Current (test function)

This version of the equivalent circuit is formed by a field excitation source  $E_0 = V_0 f_0$ , an impedance operator and a current virtual source  $J_e$  defined in metallic regions.  $J_e$  is the problem unknown approximated as a series of known test functions  $g_p$  weighted by unknown coefficients  $x_p$   $p = 1, 2, \dots, N_e$ ,

$$J_e = \sum_p g_p x_p$$

Application of Ohm and Kirchhoff laws: 
$$\begin{pmatrix} J_0 \\ E_e \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & \hat{Z} \end{pmatrix} \begin{pmatrix} E_0 \\ J_e \end{pmatrix}$$

Galrekin Method: 
$$\begin{pmatrix} I_0 \\ 0 \end{pmatrix} = \begin{bmatrix} 0 & -\langle f_0 | g_p \rangle \\ \langle g_q | f_0 \rangle & \langle g_q | \hat{Z} | g_p \rangle \end{bmatrix} \begin{bmatrix} V_0 \\ x_p \end{bmatrix} \Leftrightarrow \begin{pmatrix} I_0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 & -A \\ A^T & B \end{pmatrix} \begin{pmatrix} V_0 \\ x_p \end{pmatrix}$$

The resolution of this system allows to calculate the input impedance:

$$Z_{in} = \frac{V_0}{I_0} = \frac{1}{A^T B^{-1} A}$$

# Third GEC configuration:

## Field (source)/Field (test function)

This version of the equivalent circuit is formed by a field excitation source  $E_0 = V_0 f_0$ , an admittance operator and a field virtual source  $E_e$  defined on the aperture domain of the discontinuity surface.  $E_e$  is the problem unknown approximated as a series of known test functions  $g_p$  weighted by unknown coefficients  $v_p$   $p = 1, 2, \dots, N_e$ ,  $E_e = \sum_p g_p v_p$

Application of Ohm and Kirchhoff laws: 
$$\begin{pmatrix} J_0 \\ J_e \end{pmatrix} = \begin{pmatrix} \hat{Z}^{-1} & -\hat{Z}^{-1} \\ -\hat{Z}^{-1} & \hat{Z}^{-1} \end{pmatrix} \begin{pmatrix} E_0 \\ E_e \end{pmatrix}$$

In this case, the Galerkin method is not applicable because of the irregularity of the impedance operator. Indeed, the operator  $\hat{Z}$  doesn't contain the contribution of the fundamental mode  $TE_{10}$ . In this way, the modal basis that constitutes this operator is not complete and it is not invertible. Hence, the operator  $\hat{Z}^{-1}$  is not defined and a problems of badly scaled matrices occurs. The input impedance is not defined



# Fourth GEC configuration:

## Current (source)/Current (test function)

This version of the equivalent circuit is formed by a current excitation source  $J_0 = I_0 f_0$ , an impedance operator and a current virtual source  $J_e$  defined in metallic regions.  $J_e$  is the problem unknown approximated as a series of known test functions  $g_p$  weighted by unknown coefficients  $x_p$   $p = 1, 2, \dots, N_e$ ,

$$J_e = \sum_p g_p x_p$$

Application of Ohm and Kirchhoff laws: 
$$\begin{pmatrix} E_0 \\ E_e \end{pmatrix} = \begin{pmatrix} \hat{Y}^{-1} & \hat{Y}^{-1} \\ \hat{Y}^{-1} & \hat{Y}^{-1} \end{pmatrix} \begin{pmatrix} J_0 \\ J_e \end{pmatrix}$$

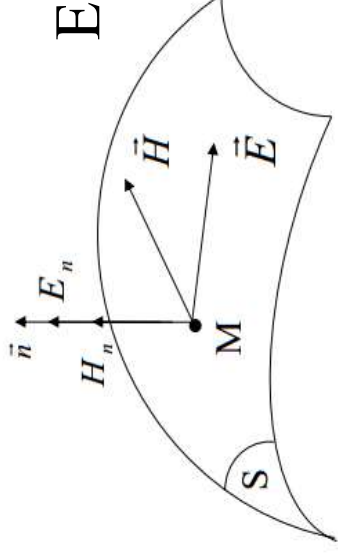
In this case, the Galerkin method is not applicable because of the irregularity of the admittance operator. Indeed, the operator  $\hat{Y}$  doesn't contain the contribution of the fundamental mode  $TE_{10}$ . In this way, the modal basis that constitutes this operator is not complete and it is not invertible. Hence, the operator  $\hat{Z}^{-1}$  is not defined and a problem of badly scaled matrices occurs. Which implies that the input impedance is not defined.

# Numerical problems

The application of the various GEC configurations provides to obtain two suitable equivalents circuit (the first and the second GEC) and two unsuitable ones (the third and the fourth GEC). These two last are unsuitable because of irregular impedance and admittance operators . In fact, these operators are not complete and undefined because of the excitation mode  $TE_{10}$  doesn't contribute in the construction of the considered operators. This leads to obtain singularities saw the no invertible operators  $\hat{Z}^{-1}$  and  $\hat{Y}^{-1}$  . So, we can't compute the input impedance and check boundary conditions when using these two configurations. To overcome this problem corresponding to the two last GEC configurations, we propose to get back the excitation mode on the impedance and admittance operators. So, this mode will not be used as excitation. This is available only when we introduce the concept of waves.

# Wave concept

We introduce the wave concept to avoid the singularity problems obtained in third and fourth equivalent circuits. In fact, this concept is based on using a wave as excitation source.



$E, H$  the tangent fields in  $M$

The wave concept is introduced by expressing the electromagnetic quantities as a function of the incident and reflected waves :

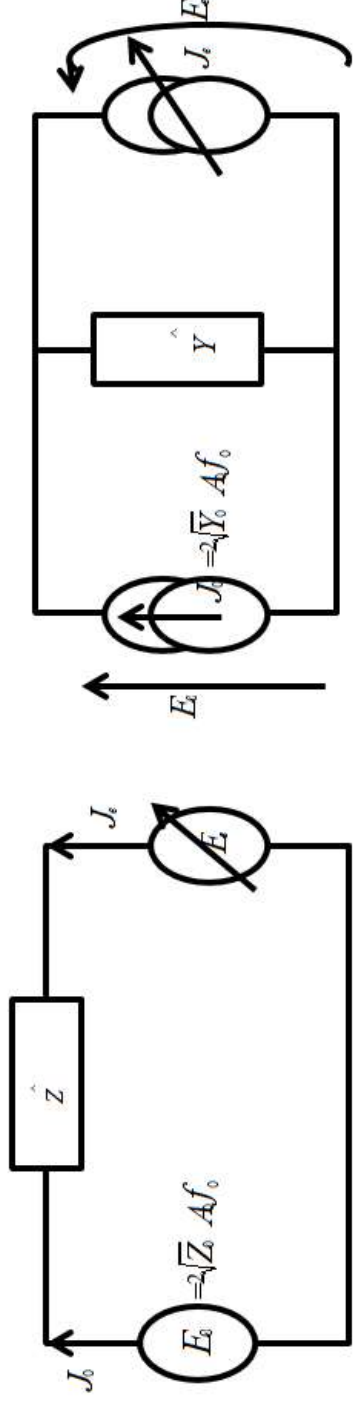
$$\begin{cases} A = \frac{1}{2\sqrt{Z_0}}(E_T + Z_0 H \wedge n) \\ B = \frac{1}{2\sqrt{Z_0}}(E - Z_0 H \wedge n) \end{cases}$$

Let  $\vec{J} = \vec{H} \wedge \vec{n}$  the current density. We obtain the equations system

$$\begin{cases} A = \frac{1}{2\sqrt{Z_0}}(E_T + Z_0 J) \\ B = \frac{1}{2\sqrt{Z_0}}(E - Z_0 J) \end{cases}$$

# Problem resolution with wave concept

We shown that it is possible to remedy them by introducing the wave concept as excitation. In fact, the solution becomes possible when using wave sources as E wave source and J wave source.



These GEC replaces the third and fourth GEC configurations; the difference here is that the fundamental mode used previously as excitation mode is introduced in Z and Y operators to describe the discontinuity. Hence, these operators are complete and don't present singularity problems

# E wave source

Generalized Ohm and Kirchhoff laws

$$\begin{cases} J_0 = \hat{Z}^{-1} E_0 - \hat{Z}^{-1} E_e \\ J_e = -\hat{Z}^{-1} E_0 + \hat{Z}^{-1} E_e \end{cases}$$

Galrekin Method: 
$$\begin{pmatrix} I_0 \\ \mathbf{0} \end{pmatrix} = \begin{pmatrix} \langle f_0 | \hat{Z}^{-1} f_0 \rangle & -\langle f_0 | \hat{Z}^{-1} g_p \rangle \\ -\langle g_q | \hat{Z}^{-1} f_0 \rangle & \langle g_q | \hat{Z}^{-1} g_p \rangle \end{pmatrix} \begin{pmatrix} V_0 \\ v_p \end{pmatrix} \Leftrightarrow \begin{pmatrix} I_0 \\ \mathbf{0} \end{pmatrix} = \begin{pmatrix} D & -A \\ -A^T & B \end{pmatrix} \begin{pmatrix} V_0 \\ v_p \end{pmatrix}$$

Input impedance: 
$$\begin{cases} I_0 = DV_0 - Av_p \\ \mathbf{0} = -A^T V_0 + Bv_p \end{cases} \Leftrightarrow Z_{in} = \frac{1}{D - A^T B^{-1} A}$$

# J wave source

Generalized Ohm and Kirchhoff laws

$$\begin{cases} E_0 = \hat{Y}^{-1} J_0 + \hat{Y}^{-1} J_e \\ E_e = \hat{Y}^{-1} J_0 + \hat{Y}^{-1} J_e \end{cases}$$

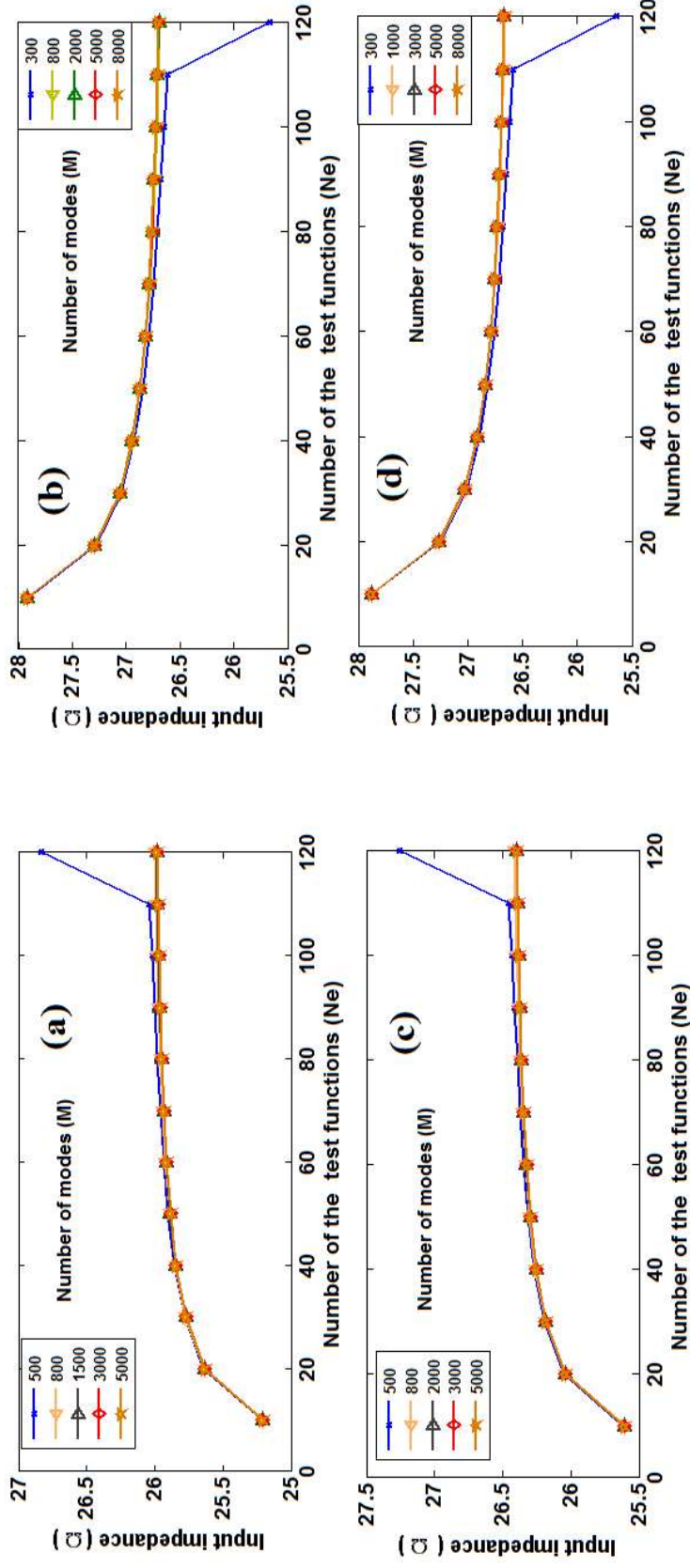
$$\begin{pmatrix} V_0 \\ \mathbf{0} \end{pmatrix} = \begin{pmatrix} \langle f_0 | \hat{Y}^{-1} f_0 \rangle & \langle f_0 | \hat{Y}^{-1} g_p \rangle \\ \langle g_q | \hat{Y}^{-1} f_0 \rangle & \langle g_q | \hat{Y}^{-1} g_p \rangle \end{pmatrix} \begin{pmatrix} I_0 \\ x_p \end{pmatrix} \Leftrightarrow \begin{pmatrix} V_0 \\ \mathbf{0} \end{pmatrix} = \begin{pmatrix} D & A \\ A^T & B \end{pmatrix} \begin{pmatrix} I_0 \\ x_p \end{pmatrix}$$

Galrekin Method:

$$\begin{cases} V_0 = DI_0 + Ax_p \\ \mathbf{0} = A^T I_0 + Bx_p \end{cases} \Leftrightarrow Z_{in} = D - A^T B^{-1} A$$

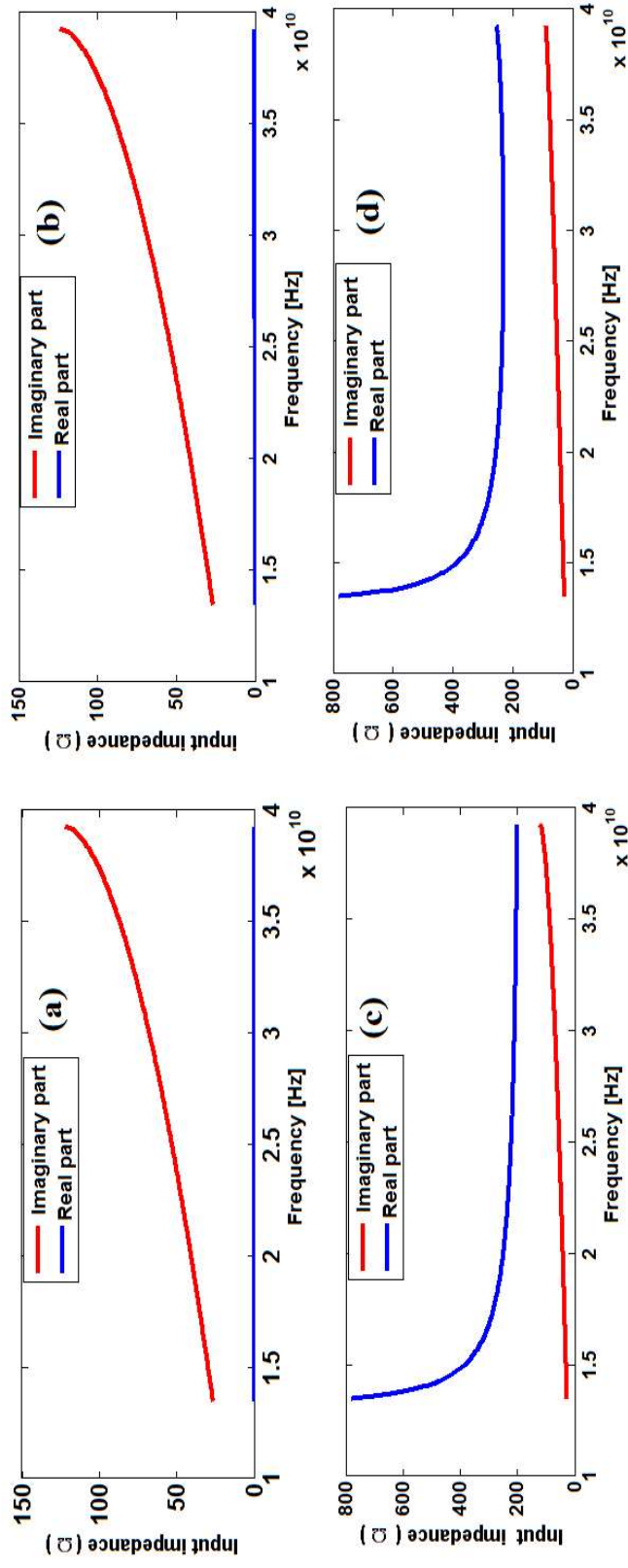
Input impedance:

# Convergence study



The convergence is given as a function of number of test functions (Ne) represented by x axis. It is given also for different number of waveguide modes(M), so we observe several curves for each configuration. It is shown that for the first and the third GEC convergence is given for about Ne=30 and M =3000 (Figure a; Figure c). However, for the second and the fourth GEC, the convergence is obtained for Ne=50 and M =5000 (Figure b; Figure d).

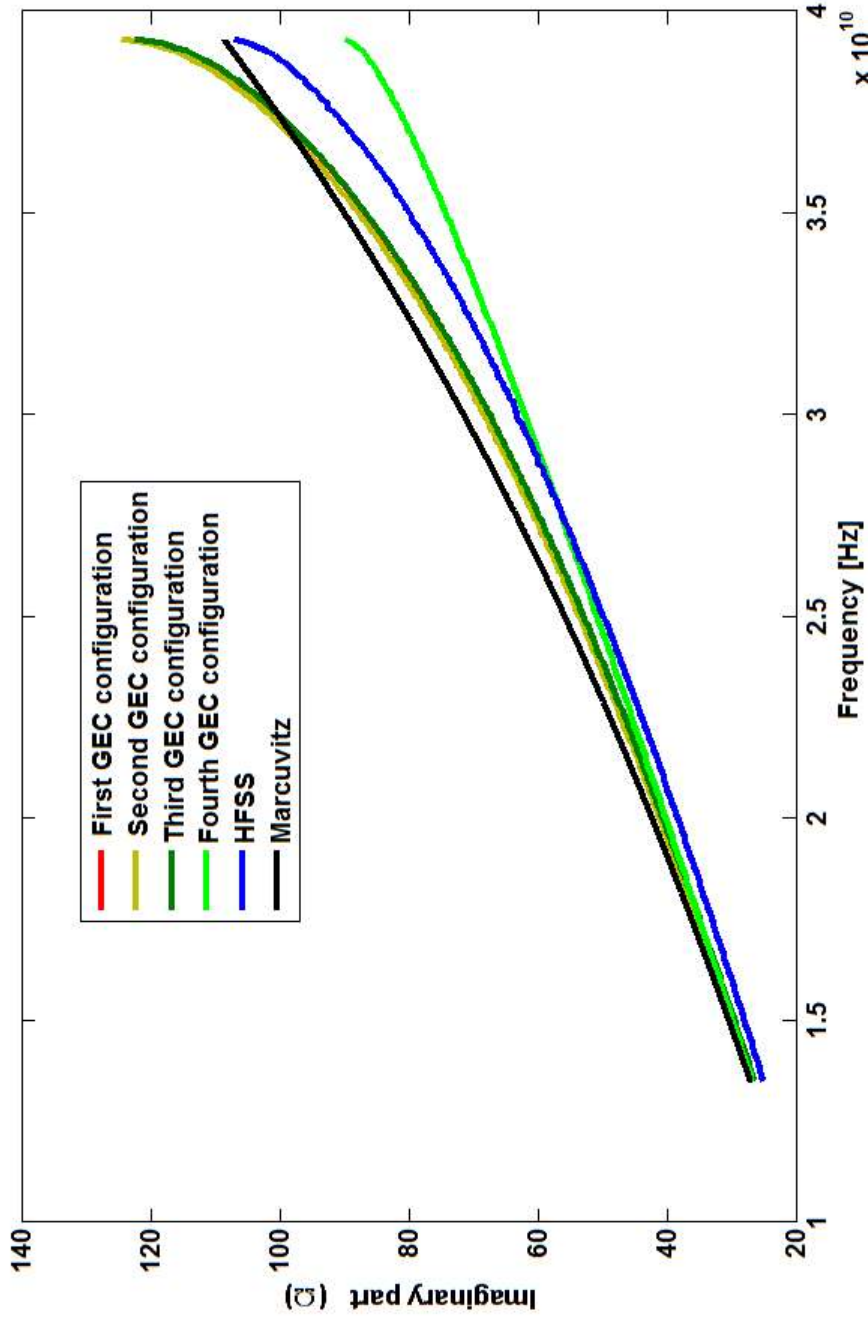
# Input impedance



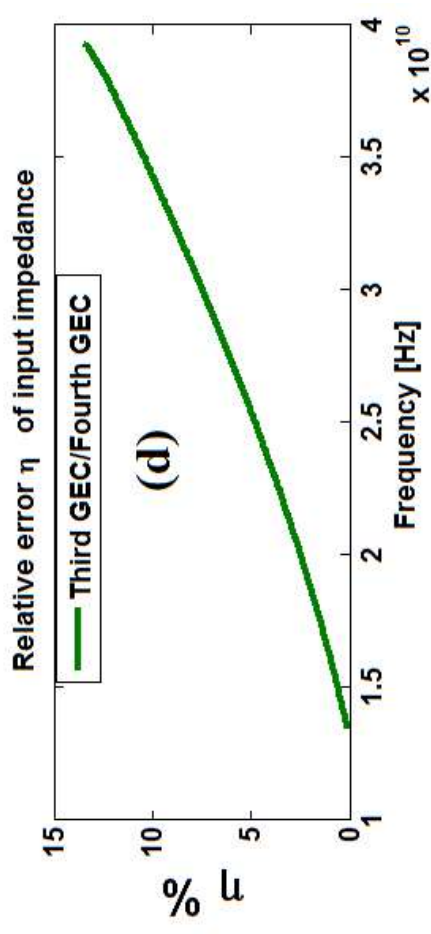
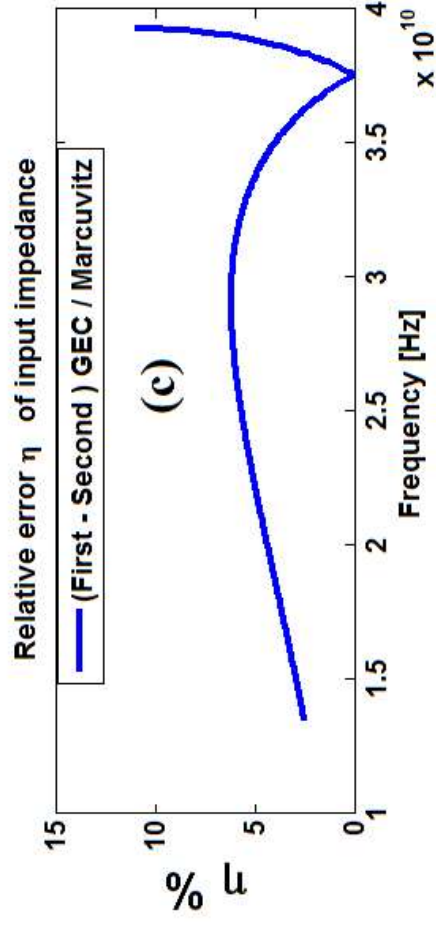
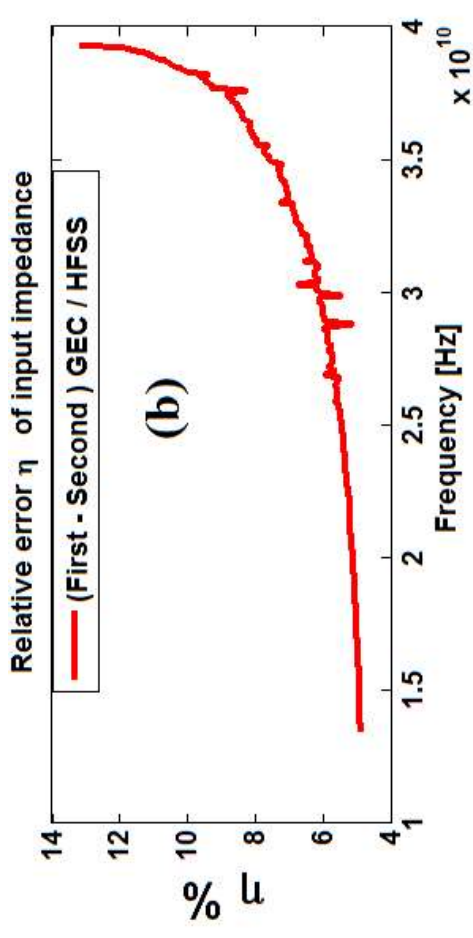
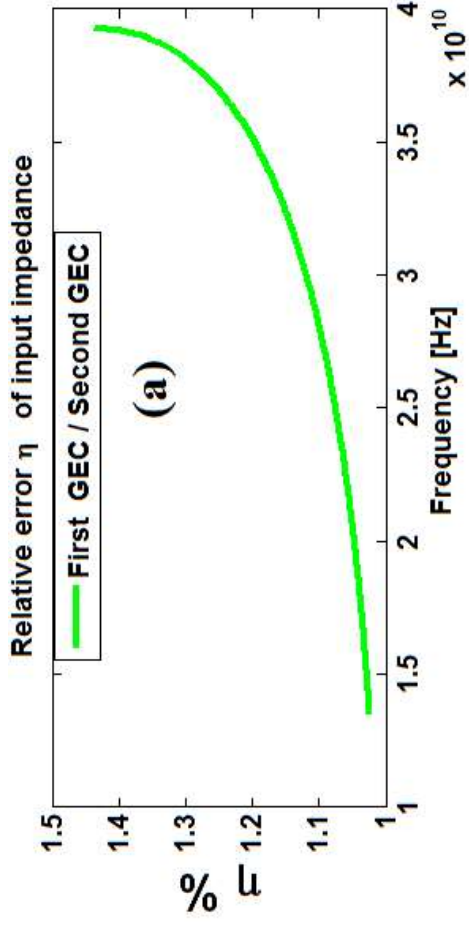
It is noted that for the cases of waveguide fundamental mode excitation (cases 1: Figure a; and cases 2: Figure b), the input impedance is purely imaginary. However, in the case of wave source excitation, (case 3: Figure c and; case 4 Figure d), the structure input impedance is complex and not purely imaginary. This is explained by the effect of the source nature, so the fundamental propagative mode is included in the impedance operator or admittance with evanescent modes.



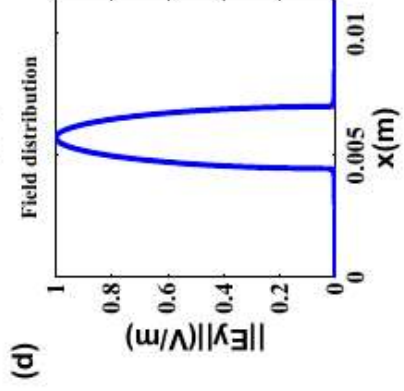
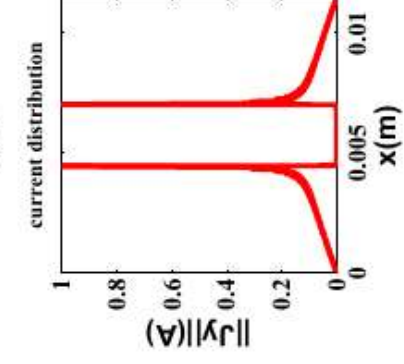
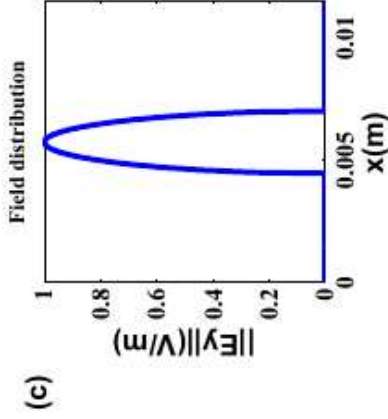
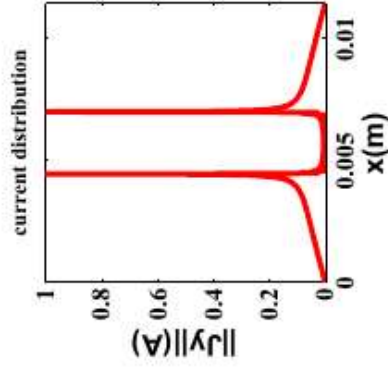
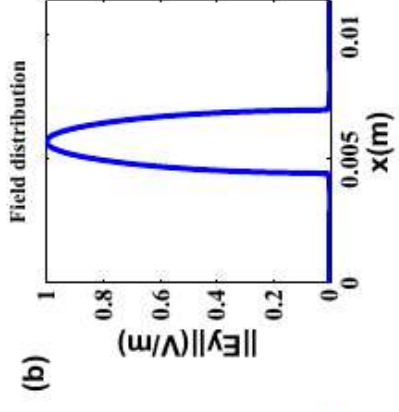
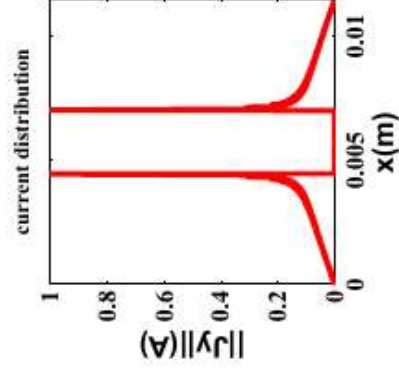
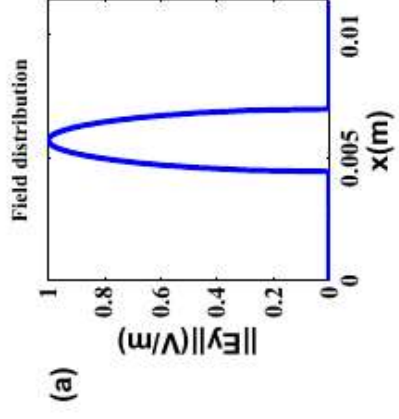
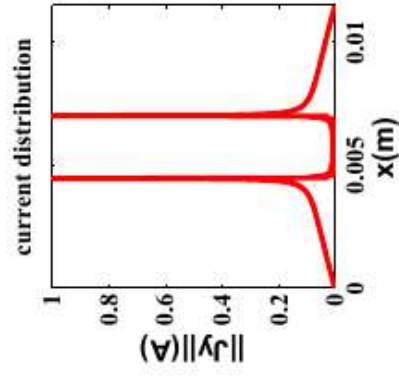
# Validation of numerical results



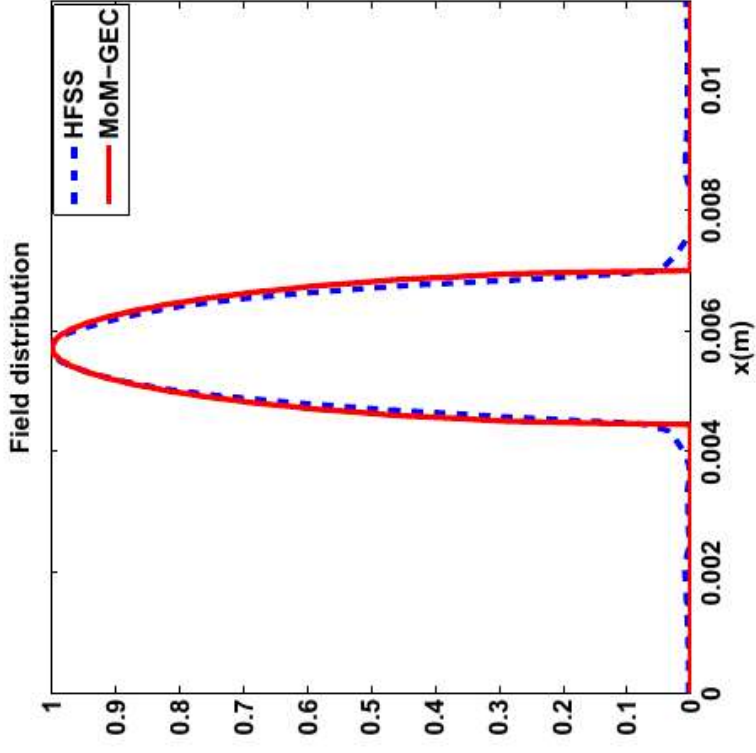
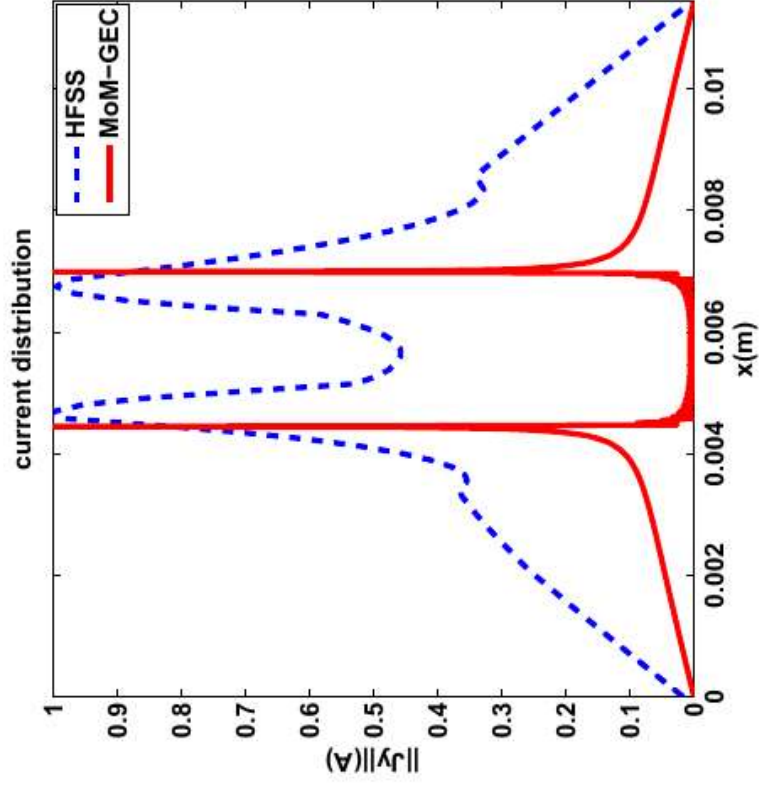
# Relative error



# Boundary conditions



# Boundary conditions-HFSS software



# Computational CPU time

Number of needed test and basis functions (Ne , M) and computational time consumed by different equivalent circuit configurations to reach the input impedance (table 1) and boundary conditions (table 2) .

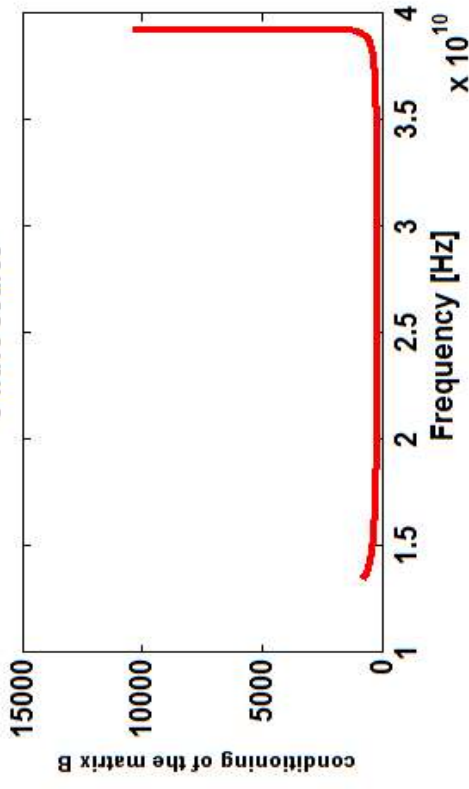
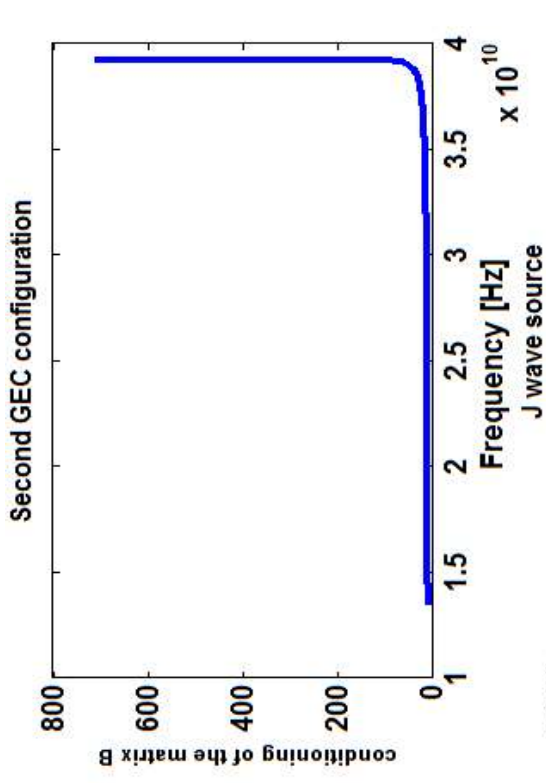
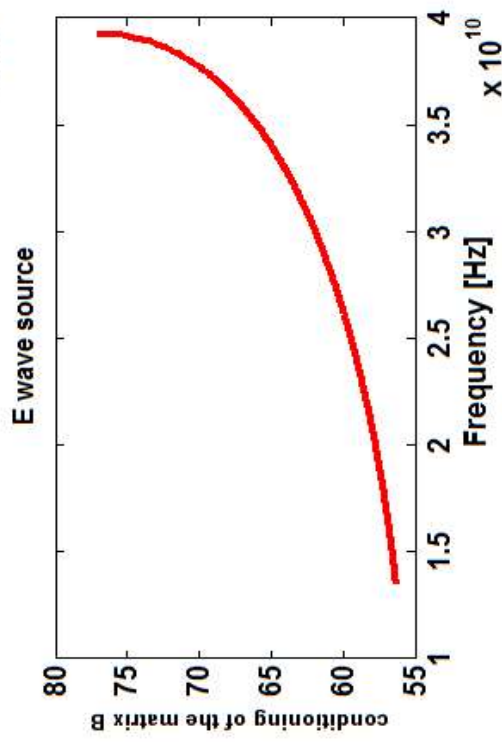
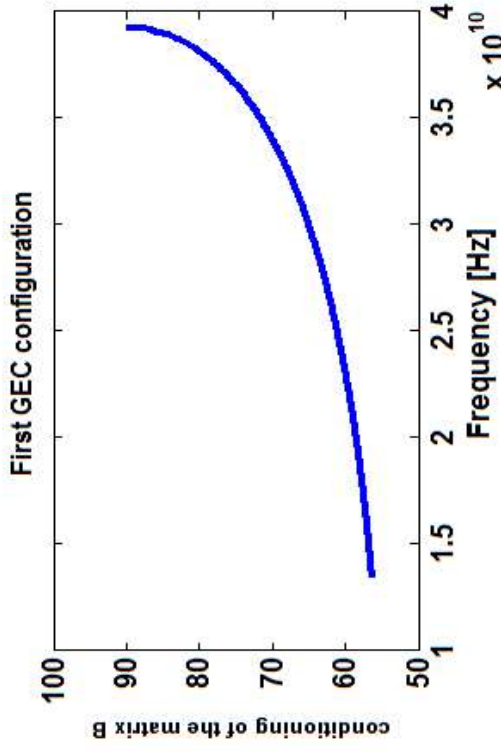
Table 1

Generalized equivalent	Basis functions M	Test functions Ne	CPU time
First equivalent circuit	3000	30	76.18 seconds
Second equivalent circuit	5000	50	317.18 seconds
Equivalent circuit (E-wave source)	3000	30	38.43 seconds
Equivalent circuit (J-wave source)	5000	50	126.35 seconds
HFSS			54.58 seconds

Table 2

Generalized equivalent circuits model	Basis functions M	Test functions Ne	CPU time
The First equivalent circuit	7000	50	5.58 seconds
The Second equivalent circuit	10000	70	12.14 seconds
Equivalent circuit (E-wave source)	7000	50	2.19 seconds
Equivalent circuit (J-wave source)	10000	70	6.72 seconds
HFSS			15.33 seconds

# Conditioning of the matrix B



# Shielding study

For studying the Shielding, we drew the variation of the coefficients of transmission according to the frequency. Indeed, by basing itself on previous results, we used the E wave operational electric circuit.

$$\Gamma = \frac{Z_{in} - Z_0}{Z_{in} + Z_0}$$

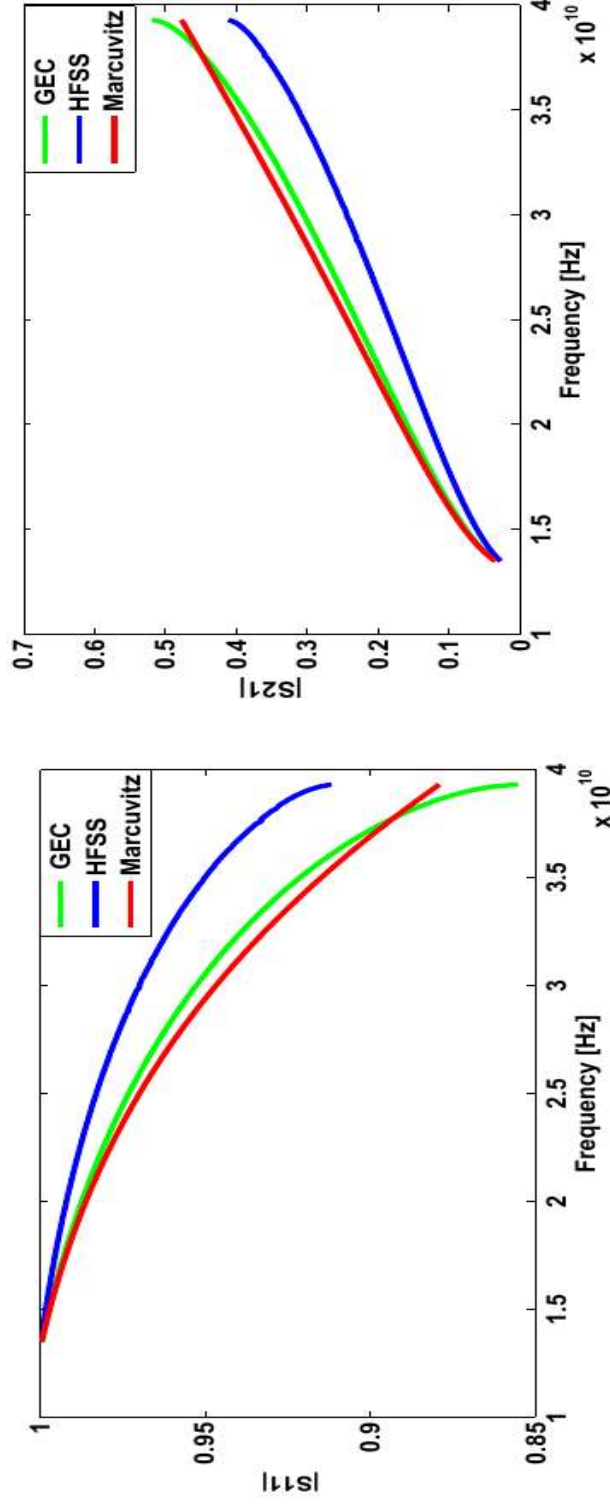


Fig. 6 Variation of parameters S11 and S21 against frequency

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